A finite element model of electric fields in the brain

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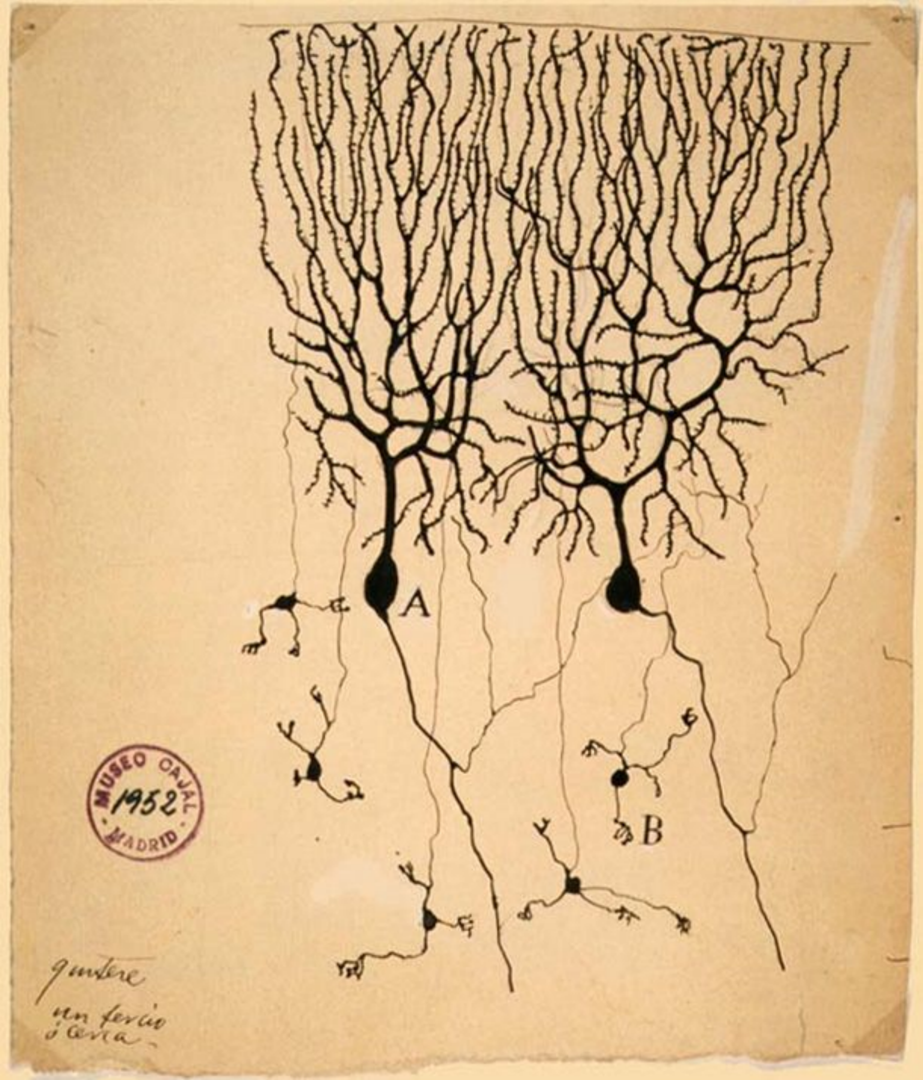
The brain is composed of billions of neurons

Neurons are cells with branching extensions which reach out and connect to other neurons



Neurons are electrically active

(sound on)

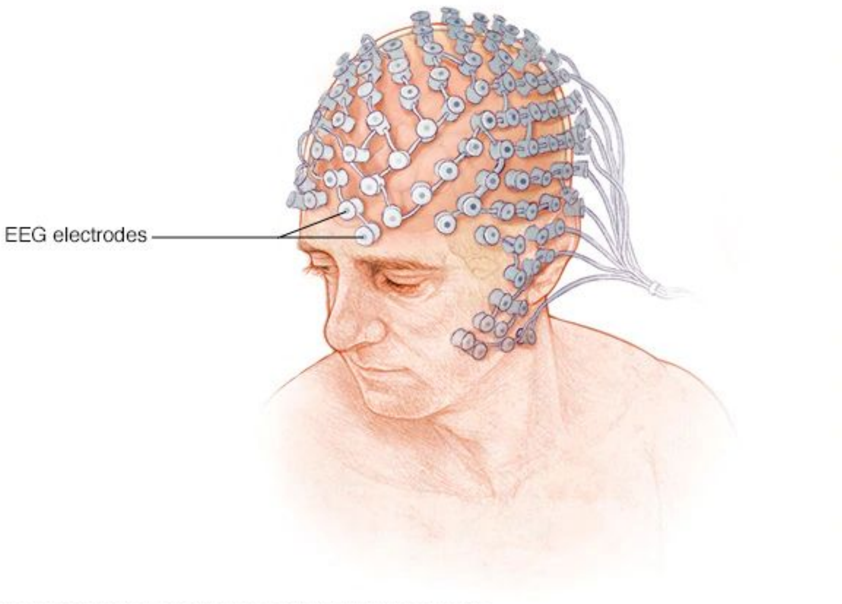
An electrode near/on a neuron *in vivo* will periodically see transient (~1ms) spikes in electrical potential

The rate of spikes usually depends on what the animal is doing, or seeing, or hearing, thinking, etc.

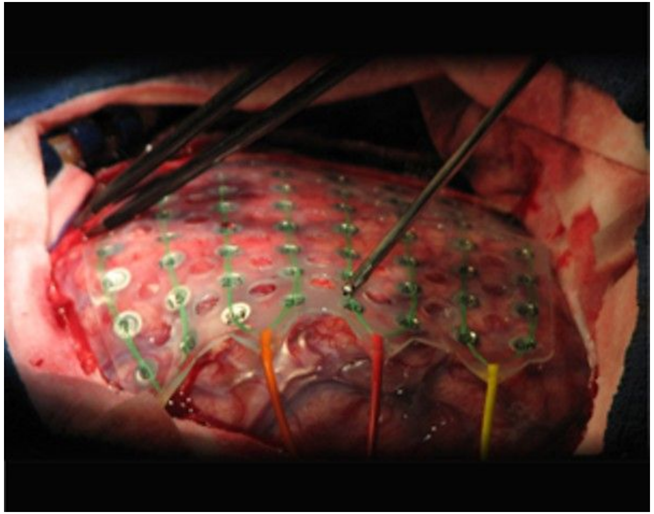
Electrode attached to speaker
Pattern drifts down/left: few spikes
Pattern drifts up/left: lots of spikes

https://www.youtube.com/watch?v=Qz40mdaDYTU

Many studies record from large populations of neurons



Electroencephalography (EEG)



Electrocorticography (ECoG)

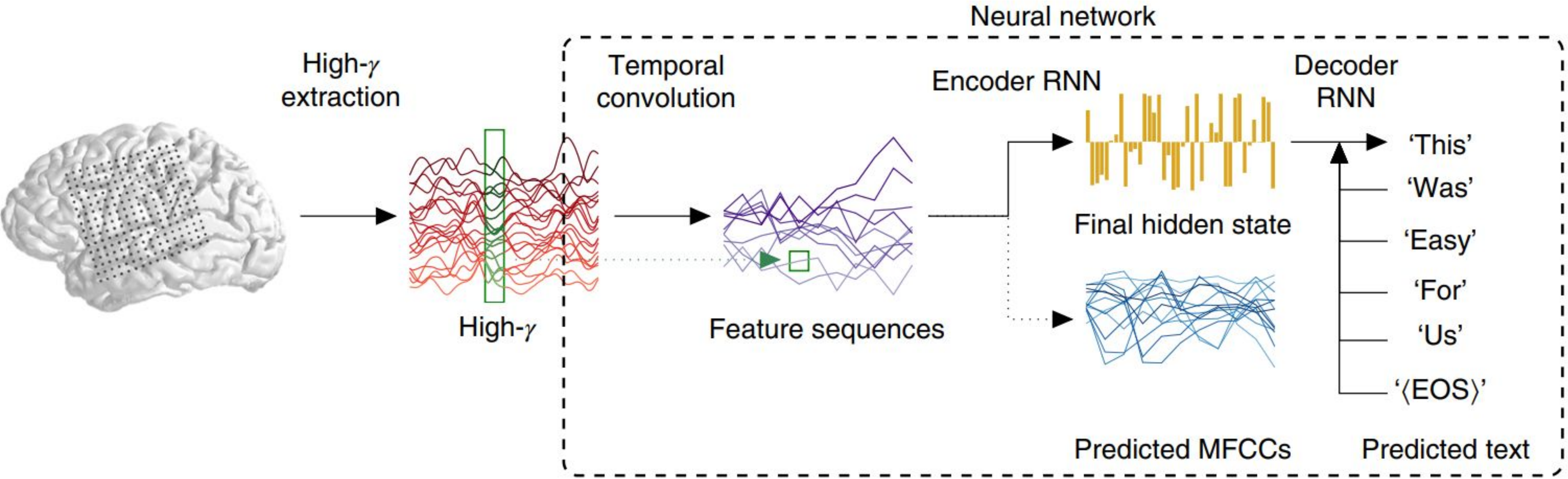
EEG and ECoG recordings reflect a superposition of the activity of $\sim 10^{5 \pm 1}$ neurons

*Population-level recordings sacrifice **resolution** in favor of **coverage***

Population-level data contains detailed information

Example: Inferring speech from neural activity

Joseph G. Makin, David A. Moses, & Edward F. Chang (2020)
https://doi.org/10.1038/s41593-020-0608-8



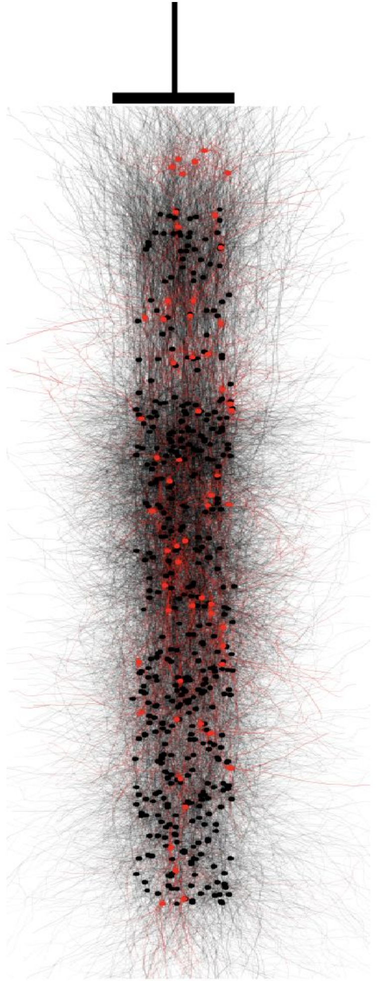
In order to understand exactly ***how*** this information is represented in the brain, we need to "invert" the population-level signal to reconstruct the activity of the underlying neuronal sources. **A detailed *forward model* may help.**

EEG and ECoG recordings reflect a superposition of signals from all nearby neurons

In order to understand exactly how information is represented in the brain, we need to “invert” the population-level signal to reconstruct the activity of the underlying neuronal sources.

This inverse problem is ill-posed: there may be different distributions of source activity which give rise to the same observed signal

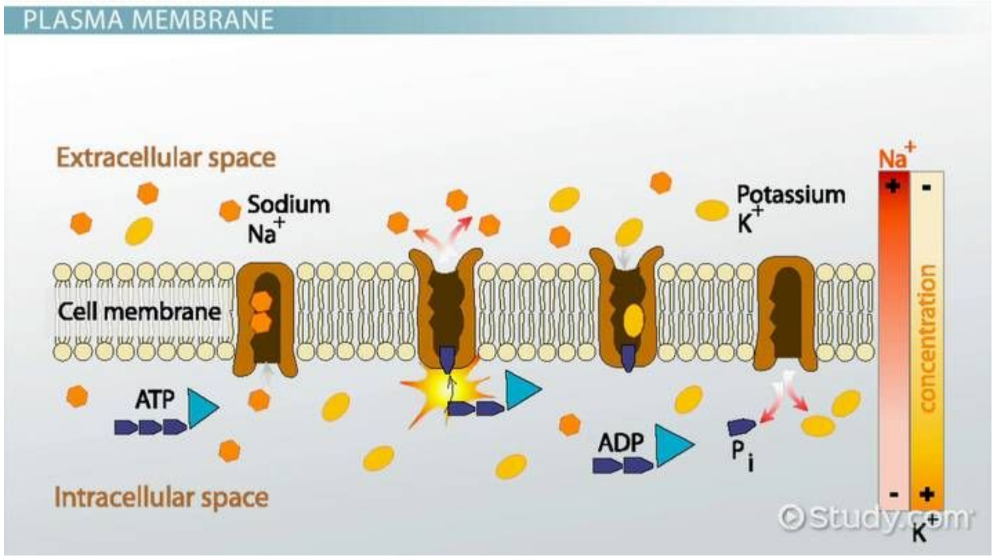
A detailed *forward model* may provide insight into which of these distributions is consistent with biology.

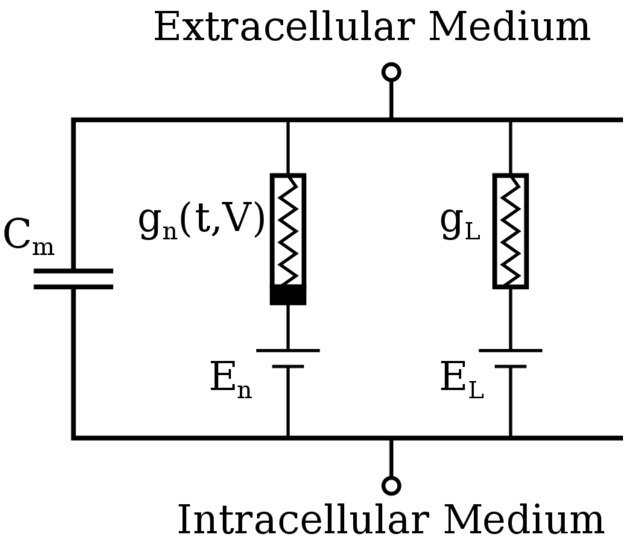


Simulating the brain

Models of neural activity are precise, accurate

1. Hodgkin & Huxley (1952) show that cell membranes behave like electrical circuits





Lipid bilayer = capacitor   Ion channels = resistors*   Electrochemical gradients = batteries

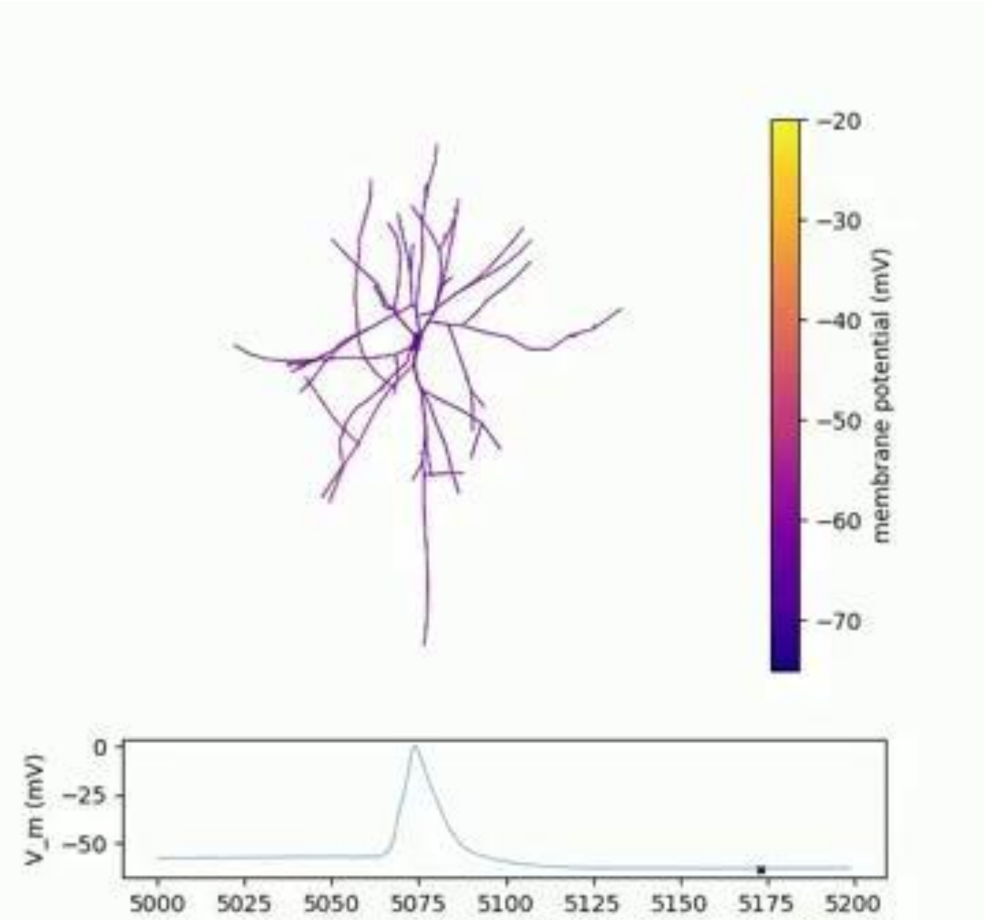
2. Cable equation describes spread of electrical potentials through neurons:

$$\frac{\partial V}{\partial T} = \frac{\partial^2 V}{\partial X} - V$$

Models of neural activity are precise, accurate

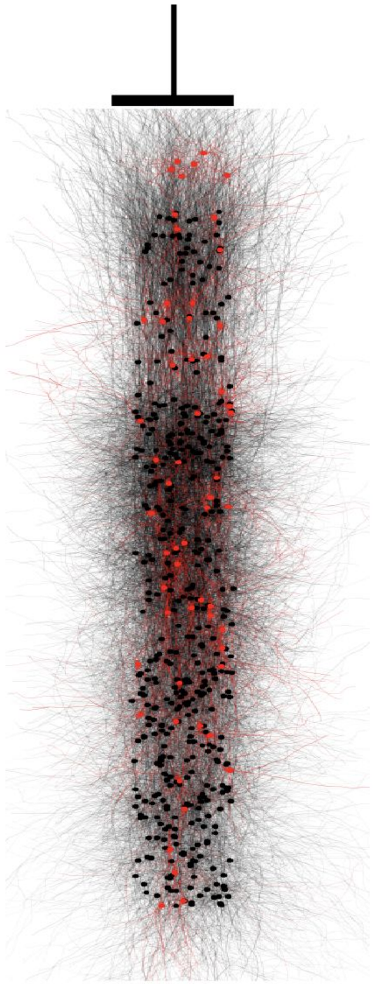
Computers can accurately simulate neural activity:

Color indicates difference in electrical potential between the inside and outside of the cell



We can simulate the *activity of neurons* in a chunk of brain.

Next: **what would an electrode near these neurons read?**



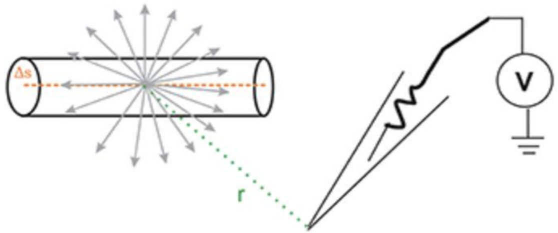
Models of extracellular potential are drastically simplified

Given an electrical current I(t) through one segment of a neuronal membrane, what signal V(**r**, t) would an external electrode located at point **r** read?

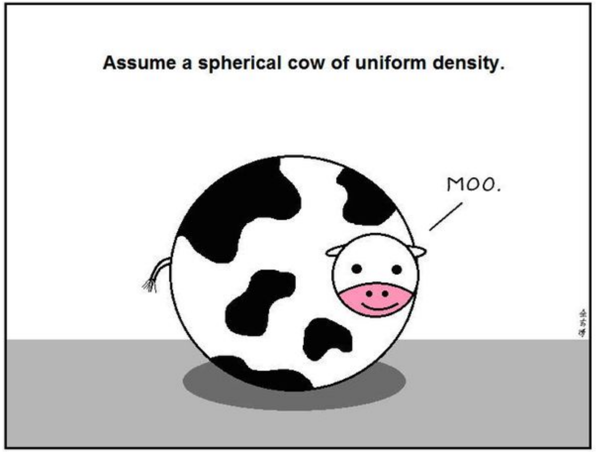
Assume extracellular space is:

- Homogeneous
- Isotropic
- Purely Ohmic (no capacitance)

Point Source Approximation

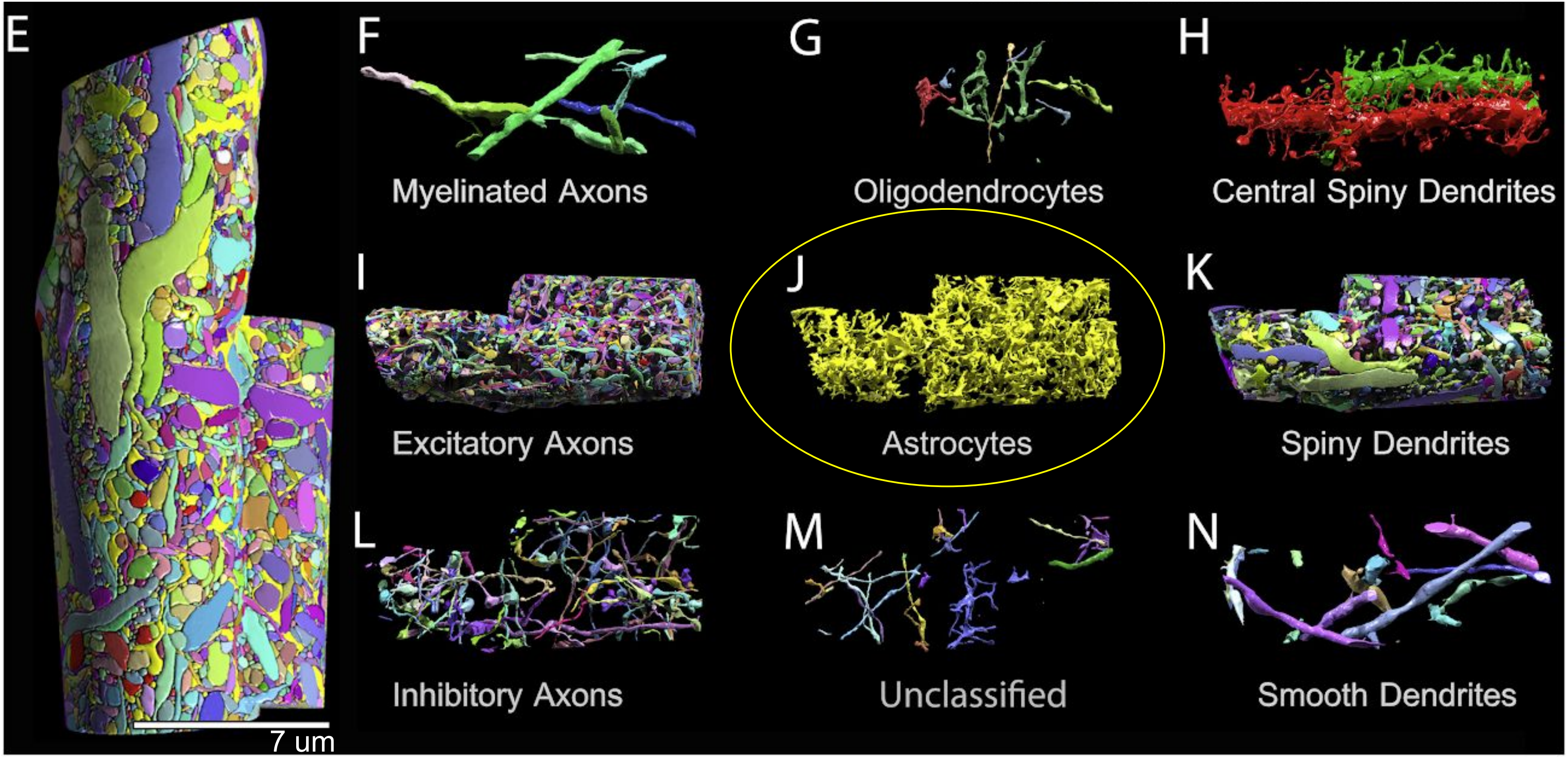


$$V(\mathbf{r},t) = \frac{I(t)}{4\pi\sigma r}$$



The extracellular medium is inhomogenous

Kasthuri, Narayanan et al.
Cell, Volume 162, Issue 3, 648 - 661



FEM handles full complexity of extracellular space

Alessio Paolo Buccino et al 2019
J. Neural Eng. 16 026030

$$\nabla \cdot \sigma_i \nabla u_i = 0 \quad \text{in } \Omega_i, \quad (1)$$

$$\nabla \cdot \sigma_e \nabla u_e = 0 \quad \text{in } \Omega_e, \quad (2)$$

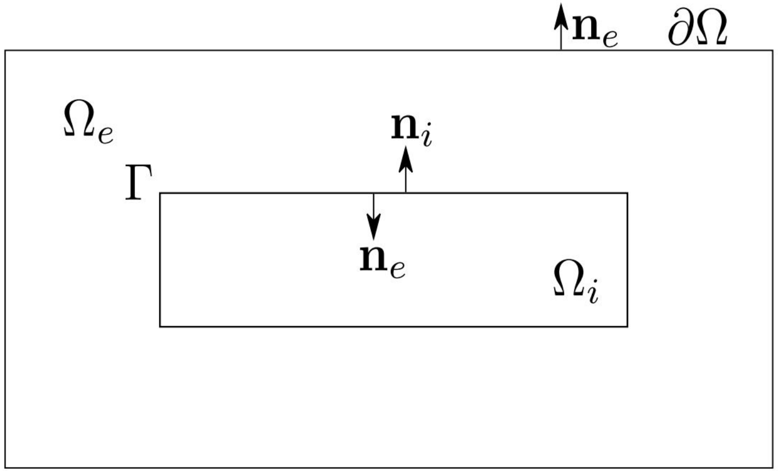
$$u_e = 0 \quad \text{at } \partial\Omega_e, \quad (3)$$

$$\sigma_e \nabla u_e \cdot n_e = 0 \quad \text{at } \partial\Omega_p, \quad (4)$$

$$n_e \cdot \sigma_e \nabla u_e = -n_i \cdot \sigma_i \nabla u_i \stackrel{\text{def}}{=} I_{\text{m}} \quad \text{at } \Gamma, \quad (5)$$

$$u_i - u_e = v \quad \text{at } \Gamma, \quad (6)$$

$$\frac{\partial v}{\partial t} = \frac{1}{C_m}(I_{\text{m}} - I_{\text{ion}}) \quad \text{at } \Gamma. \quad (7)$$



Interior and exterior of cells are separate, but coupled by the membrane (6)

(7) gives the time evolution of the membrane potential

Overview

(Population recordings)

Electrical input → Neural activity → Extracellular potential → “Useful” information

Hodgkin-Huxley membrane model, cable equation simulations

~~Point source approximation~~

FEM

Principal Components Analysis, Neural Networks, etc.