FEniCS 2021

Stochastic topology optimisation for robust and manufacturable designs

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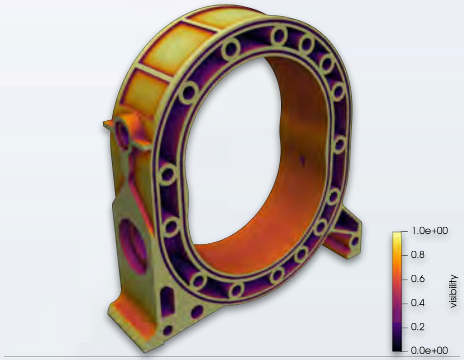
About Rafinex

- spinoff from the **Weierstrass Institute** and **Imperial College**
- based in **Luxembourg**, UK and Germany
- **Université du Luxembourg** research cooperation → *Martin Řehoř's talk*
- algorithms development for
  - robustness analysis
  - robust topology optimisation
  - manufacturability and costing prediction
  - special purpose optimisation
- sectors: automotive, aerospace, tooling, biomedical, consumer electronics

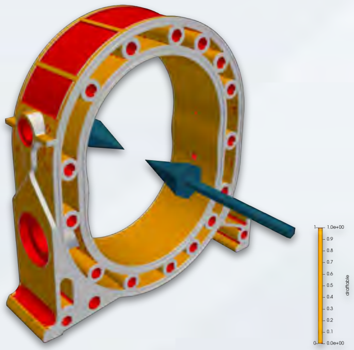
What is a good design?

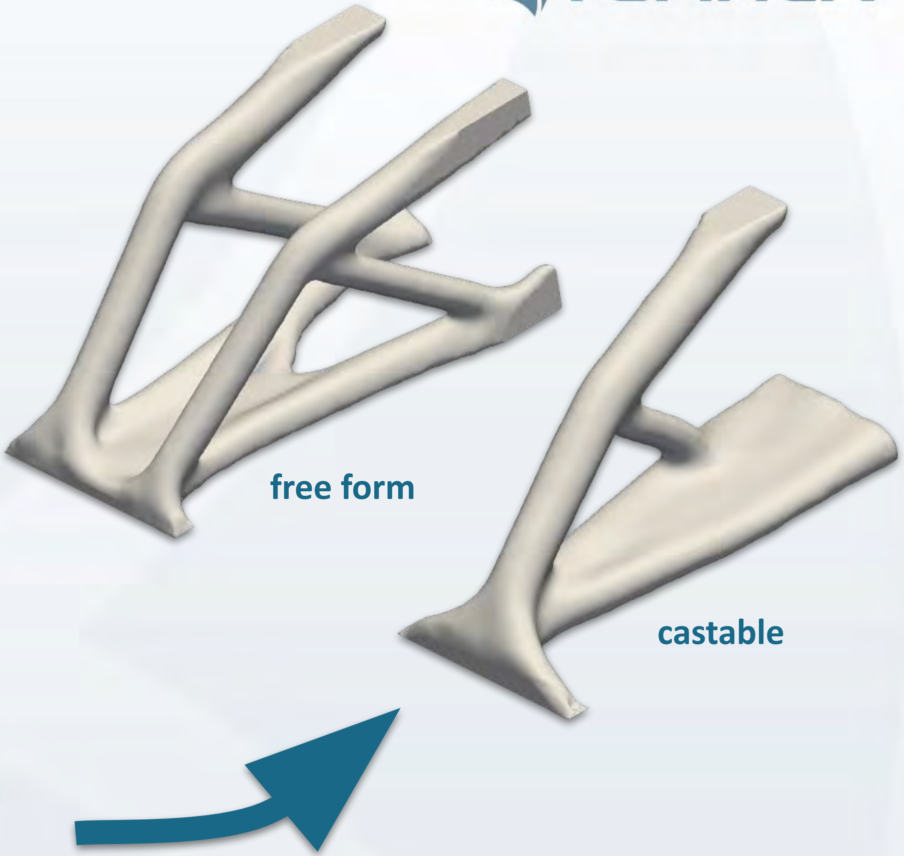
- high performance (e.g. stiffness)
- lightweight
- easy to manufacture
- ...

reachability analysis for CNC

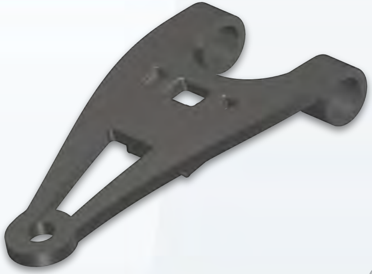


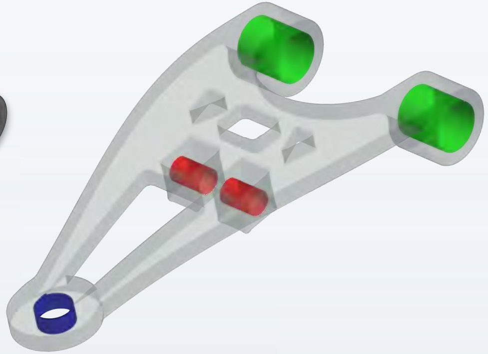
castability analysis

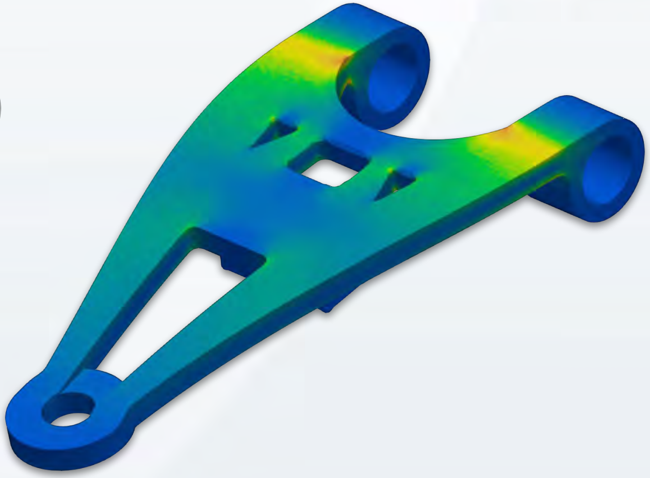




Evaluation of Designs







- gain insights on designs
- allows us to compare designs
- allows us to modify the design

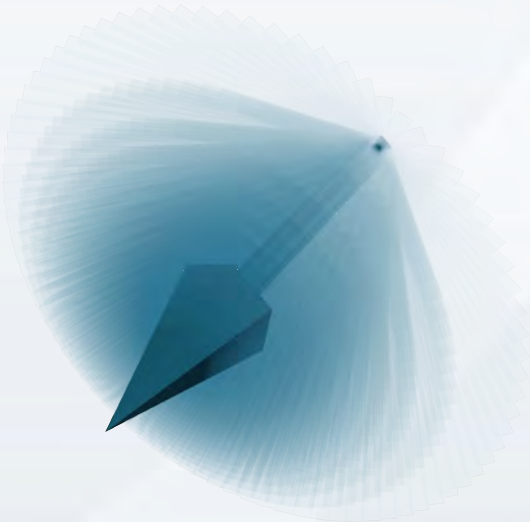
randomize + compare = generative design

$$
\begin{aligned}
-\operatorname{div}\left[\mathcal{C}(\varphi)\mathcal{E}(u)\right] &= \varphi f && \text{in } D,\\
u &= 0 && \text{on } \Gamma_d\\
\left[\mathcal{C}(\varphi)\mathcal{E}(u)\right]\cdot n &= g && \text{on } \Gamma_g\\
u \cdot n &= 0 && \text{on } \Gamma_s\\
\left[\mathcal{C}(\varphi)\mathcal{E}(u)\right]\cdot n &= 0 && \text{on } \Gamma_0 = \partial D \setminus (\Gamma_d \cup \Gamma_g \cup \Gamma_s)
\end{aligned}
$$

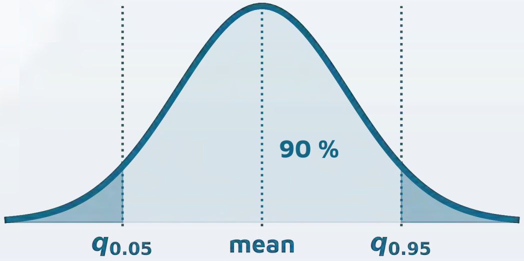
see the FEniCS tutorials → Linear Elasticity

Modelling Variability

Deterministic
load vector

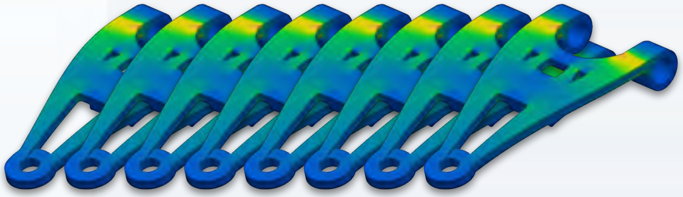


Stochastic
load vector

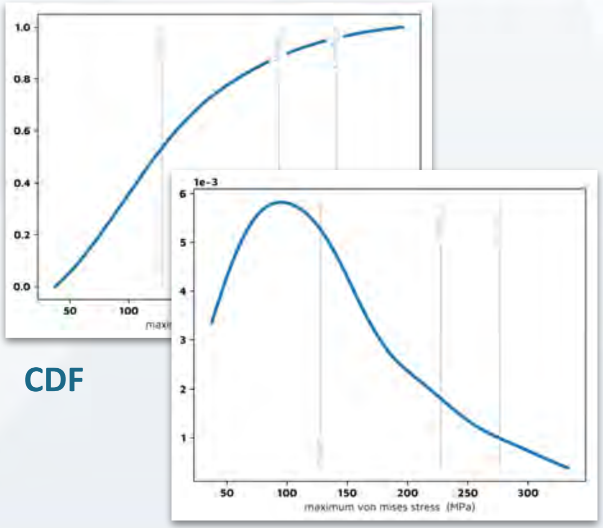


Stochastic load cases
are defined by PDFs

Evaluating Stochastic Results



Scalars



CDF

PDF

Fields
Scalars

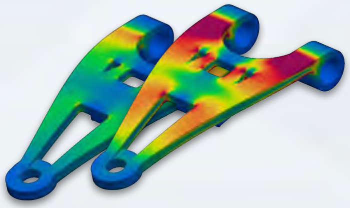
Mean

Variance

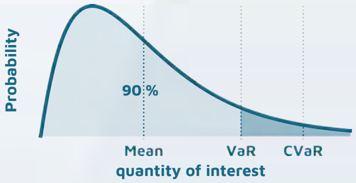
Standard Deviation

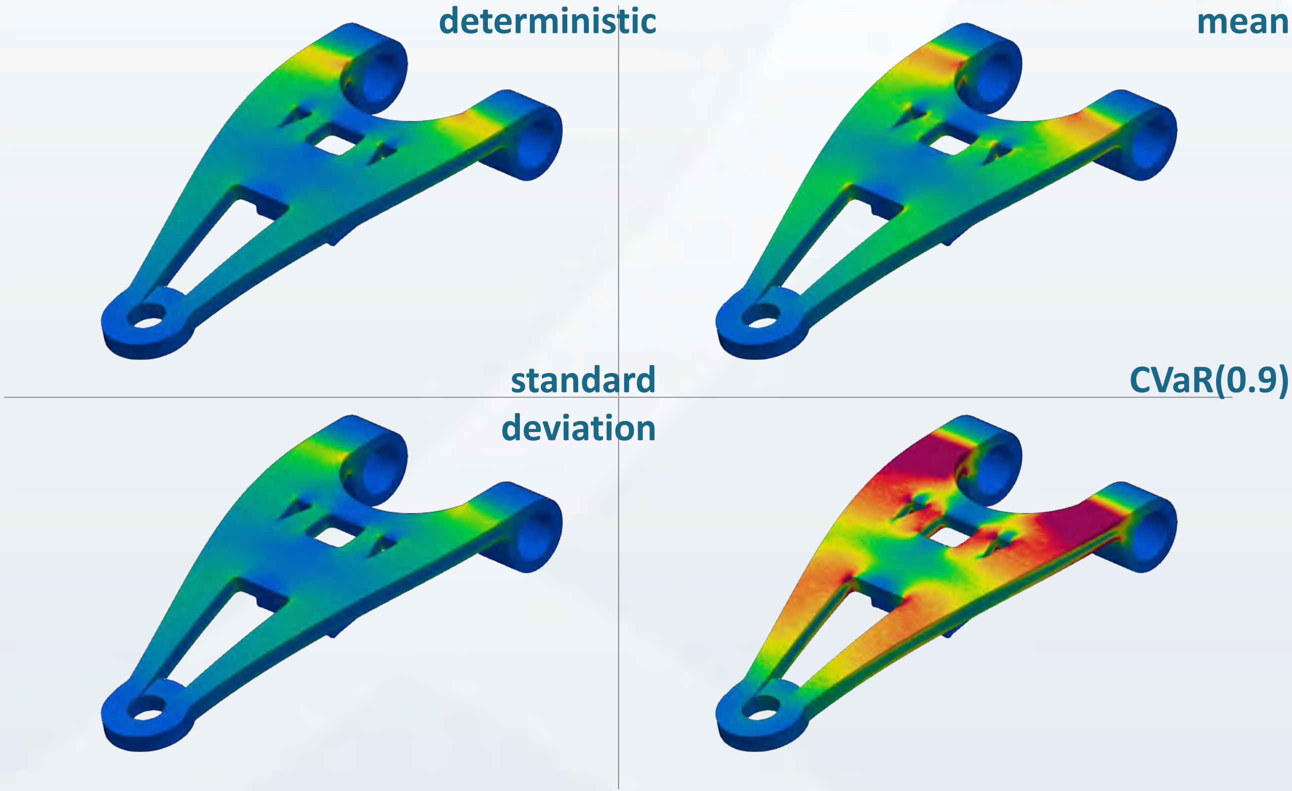
Value at Risk (VaR)

Conditional Value at Risk (CVaR)



Field Data


Probability
90 %
Mean
VaR
CVaR
quantity of interest

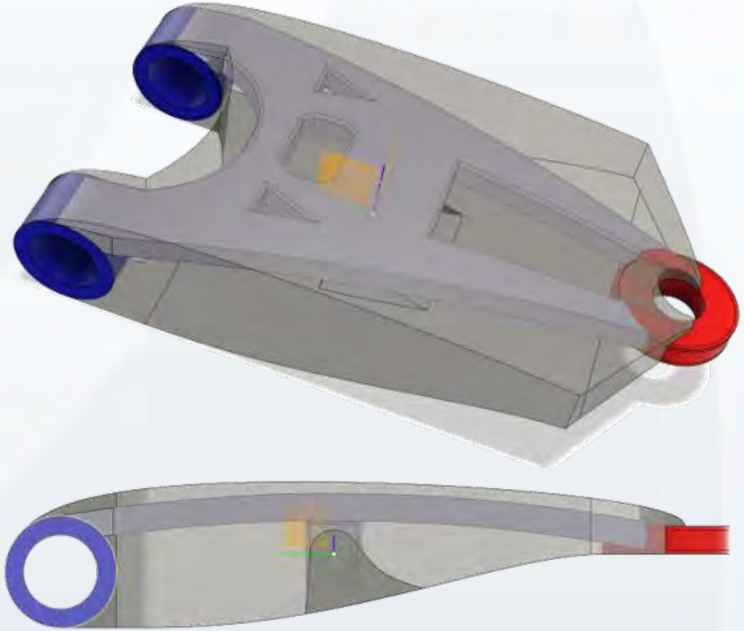

deterministic
mean
standard deviation
CVaR(0.9)

Topology Optimisation

gradient from the target functional via the adjoint method

user defines envelope/constraints

- available build space
- performance/acceptance criteria (max. stress, max. displacement)
- manufacturing process

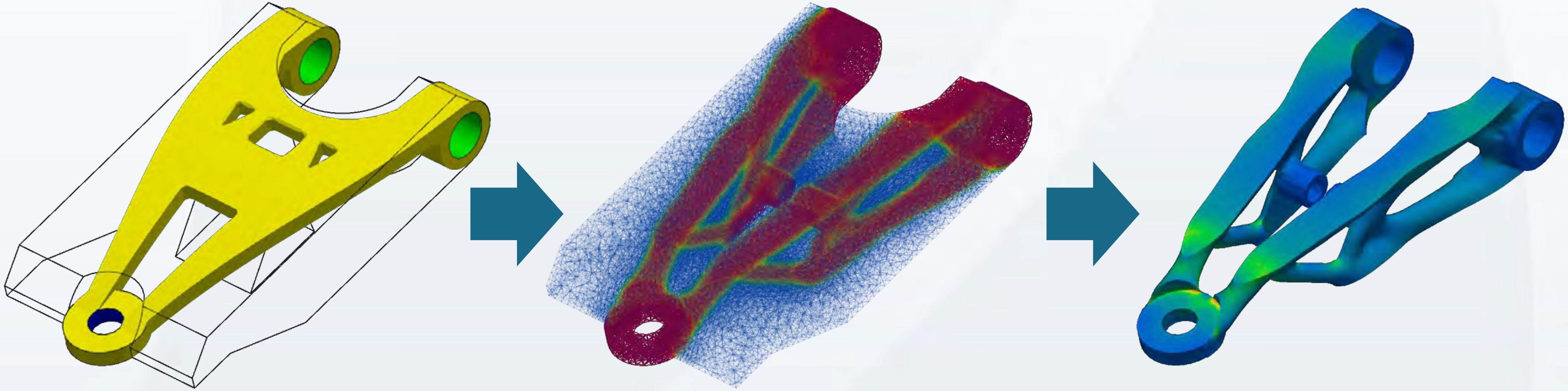


$$0 = \underbrace{\frac{\varepsilon}{\tau}\int_D (\varphi - \varphi_n) v_\varphi \,\mathrm{d}x}_{\text{gradient step } \tau} + \underbrace{\varepsilon\gamma \int_D \nabla\varphi \cdot \nabla v_\varphi \,\mathrm{d}x + \frac{\gamma}{\varepsilon}\int_D \frac{\partial \psi_0}{\partial \varphi}(\varphi_n) v_\varphi \,\mathrm{d}x}_{\text{interface gradient}}$$

$$- \underbrace{\int_D \mathcal{E}(p(\varphi)) : \frac{\partial \mathcal{C}}{\partial \varphi} v_\varphi \mathcal{E}(u(\varphi)) \,\mathrm{d}x - \int_D f \cdot p v_\varphi \,\mathrm{d}x}_{\text{state gradient}} + \underbrace{\int_D (\varphi - m) v_\lambda \,\mathrm{d}x + \int_D \lambda v_\varphi \,\mathrm{d}x}_{\text{volume lagrange multiplier}}$$

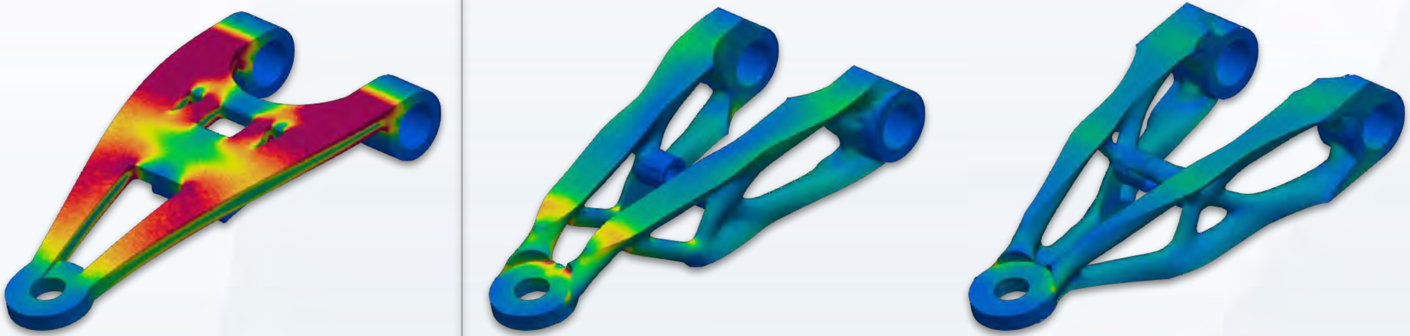
adjoint method

Topology Optimisation

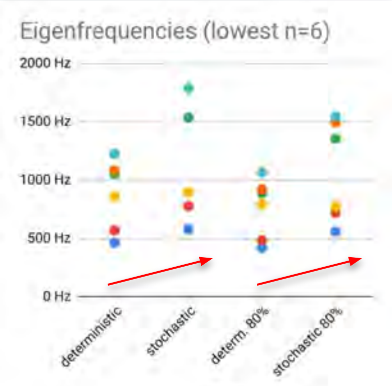


large load on purple boundary and small load in the middle, fixed at green boundary
heavy use of mesh adaptivity

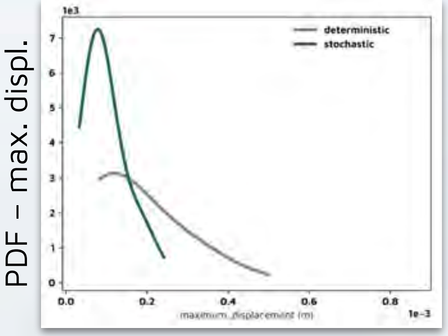
Robust Topology Optimisation

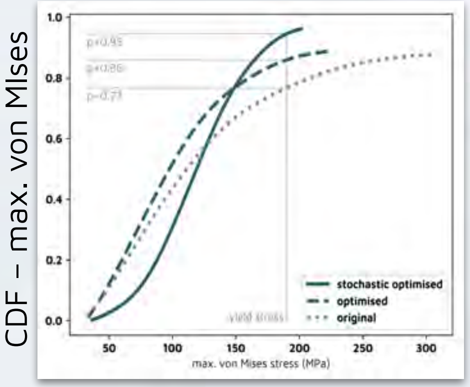


baseline | deterministic | stochastic

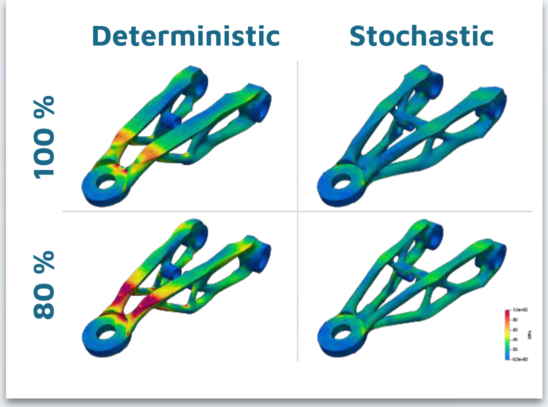


conditional value at risk at 0.9

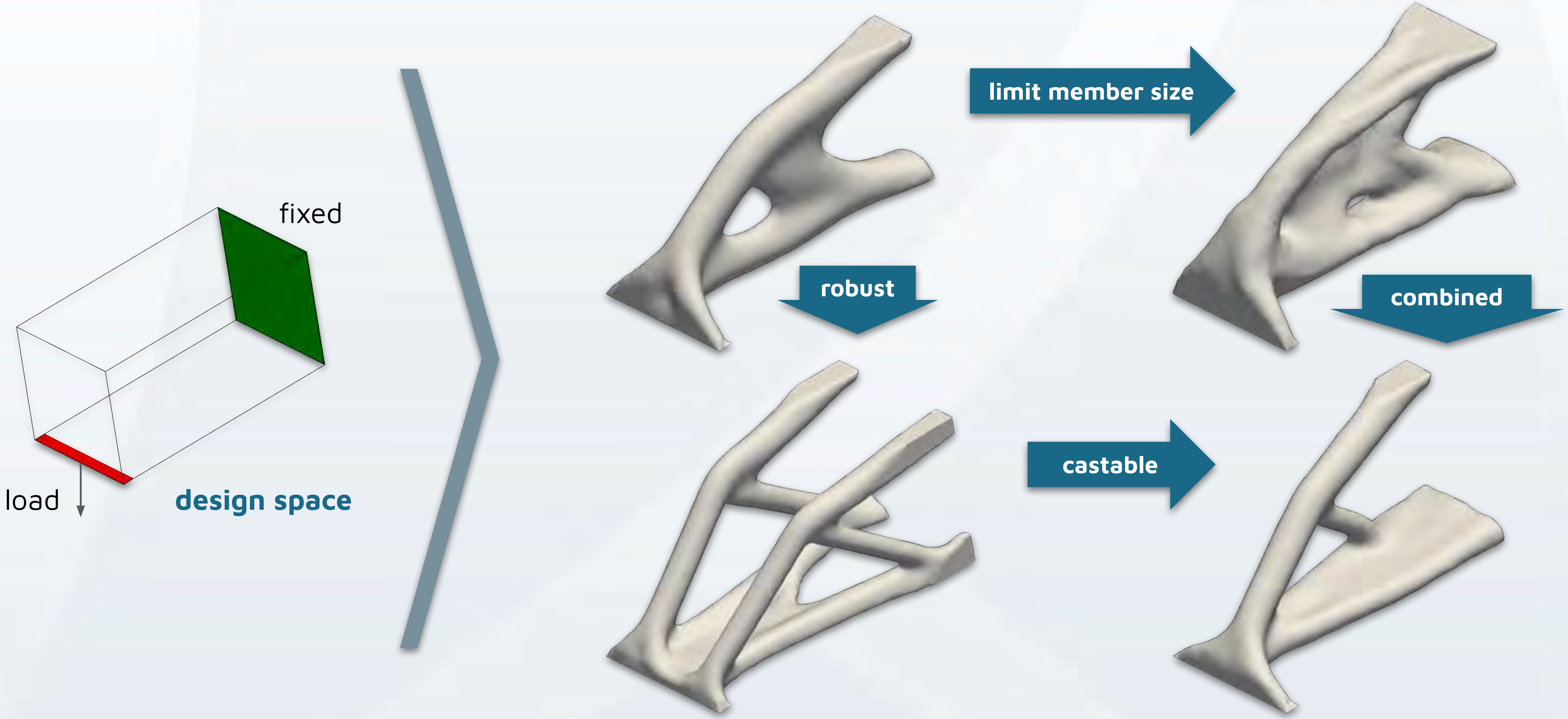




lighter
or
safer



Manufacturable Designs



3D Printed Car Optimization

rafinex

rafinex

SCALED
IDEAS MADE BIG

Thanks for your attention!